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Limiting Nusselt number for multiparticle system under dynamic condition

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INTRODUCTION

MUCH WORK has been done on the theoretical prediction of particle–fluid transfer coefficients for a multiparticle–fluid system under the steady-state condition (e.g. refs. [1–4]), but less attention has been focused on those under the dynamic condition [5]. It has usually been assumed that transfer coefficients under steady-state and dynamic conditions are the same. The purpose of this work is to examine if these particle–fluid transfer coefficients are identical. The present study is confined to a low flow rate region, and based on the free surface model [6] which has successfully been used in the studies of transfer rate in multiparticle–fluid systems [1, 3].

SPHERICAL UNIT CELL

Let us consider heat conduction in the concentric spherical cell of radii R (for solid particle) and R_o (for fluid envelope). A temperature change imposed over the outer fluid surface results in heat transfer between the particle and fluid at the particle surface. When there exists no heat source or sink within the particle, the system is described as

$$\frac{\partial T_F}{\partial t} = \alpha_F \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T_F}{\partial r}\right) \quad \text{for } R < r < R_o \tag{1}$$

$$\frac{\partial T_S}{\partial t} = \alpha_S \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T_S}{\partial r}\right) \quad \text{for } r < R \tag{2}$$

$$T_F = T_S \quad \text{and} \quad k_F \frac{\partial T_F}{\partial r} = k_S \frac{\partial T_S}{\partial r} \quad \text{at } r = R. \tag{3}$$

Suppose that a sinusoidal temperature change, T_o, is given over the outer surface of the fluid envelope:

$$T_o = T_\omega \cos \omega t \quad \text{at } r = R_o. \tag{4}$$

The asymptotic solutions for fluid and solid temperatures can be expressed, respectively, as $T_F = Re[\hat{T}_F \exp(i\omega t)]$ and $T_S = Re[\hat{T}_S \exp(i\omega t)]$, where $\hat{T}_F$ and $\hat{T}_S$ are complex amplitudes. The temperature on the particle surface and the heat flux from the fluid to the solid surface are also expressed in terms of $Re[(\hat{T}_S)_{r=R} \exp(i\omega t)]$ and $Re[k_S(\partial \hat{T}_S/\partial r)_{r=R} \times \exp(i\omega t)]$, respectively.

LIMITING NUSSELT NUMBER UNDER DYNAMIC CONDITION

A particle–fluid Nusselt number is defined as

$$Nu = \frac{2R}{k_F} \frac{k_S \left(\dfrac{\partial T_S}{\partial r}\right)_{r=R}}{T_F^* - (T_S)_{r=R}} \tag{5}$$

where T_F^* is a representative value of the fluid temperature, such as the volume-mean or cup-mixing average. The numerator and denominator on the right-hand side of equation (5) are functions of time under the dynamic condition. However, if Nu is constant, the Nusselt number can be extended under the dynamic condition.

On the other hand, the temperature change can be expressed as the sum of its harmonic components. Similar to equation (5), the limiting Nusselt number is defined for each component as

$$\widehat{Nu} = \frac{2R}{k_F} \frac{k_S \left(\dfrac{\partial \hat{T}_S}{\partial r}\right)_{r=R}}{\hat{T}_F^* - (\hat{T}_S)_{r=R}} \tag{6}$$

where $\hat{T}_F^*$ is the complex amplitude of the representative value of fluid temperature, i.e. $T_F^* = Re[\hat{T}_F^* \exp(i\omega t)]$. If $\widehat{Nu}$ is considered constant for the components, the Nusselt number is well defined under the dynamic condition. In this case the definitions based on equations (5) and (6) become equivalent.

The Nusselt number depends upon the representative value of the fluid temperature, T_F^*. Pfeffer and Happel [1] assumed the outer surface temperature to be T_F^*. Miyauchi [2] pointed out that the cup-mixing value should be taken as T_F^* even in the limiting case of zero flow rate. In any event, the representative value of fluid temperature is considered to fall between the outer surface temperature, T_o, and the volume-mean temperature, T_V.

If the limiting Nusselt number is well defined, its value is obtained by taking the limit at $\omega = 0$ of equation (6) as:

for $T_F^* = T_o$

$$Nu^0 = \frac{2}{1 - \dfrac{1}{\xi_o} + \dfrac{1}{2\gamma\xi_o}(\xi_o - 1)^2(\xi_o + 2)} \tag{7}$$

for $T_F^* = T_V$

$$Nu^0 = \frac{2}{1 - \dfrac{3}{2}\dfrac{\xi_o^2 - 1}{\xi_o^3 - 1} + \dfrac{3}{2\gamma}\left(\dfrac{1}{5}\dfrac{\xi_o^5 - 1}{\xi_o^3 - 1} + \dfrac{\xi_o^2 - 1}{\xi_o^3 - 1} - 1\right)} \tag{8}$$

where

$$\gamma = \frac{C_S \rho_S}{C_F \rho_F} \tag{8a}$$

$$\xi_o = \frac{R_o}{R}. \tag{8b}$$

The limiting Nusselt numbers given by equations (7) and (8) are shown in Fig. 1, where the fluid volume fraction, ε_b, is $1-(R/R_o)^3$.

As γ increases, Nu^0 approaches the limiting Nusselt number under the steady-state condition:

for $T_F^* = T_o$

$$Nu^0 = \frac{2}{1 - \dfrac{1}{\xi_o}} \tag{9}$$

for $T_F^* = T_V$

$$Nu^0 = \frac{2}{1 - \dfrac{3}{2}\dfrac{\xi_o^2 - 1}{\xi_o^3 - 1}}. \tag{10}$$

NOMENCLATURE

C	specific heat of fluid [J kg^{-1} K^{-1}]
K_A	adsorption equilibrium constant [m^3 kg^{-1}]
k	thermal conductivity [W m^{-1} K^{-1}]
Nu	Nusselt number
R	radius of solid particle [m]
R_o	radius of spherical cell [m]
Re	particle Reynolds number
r	radial distance variable from the center of the particle [m]
T	temperature [K]
T_ω	amplitude of sinusoidal temperature change with frequency ω imposed on the outer surface [K]
t	time [s].

Greek symbols

α	thermal diffusivity, $k/(C\rho)$ [m^2 s^{-1}]
γ	$C_S\rho_S/C_F\rho_F$ for heat transfer; $\varepsilon_p+\rho_S K_A$ for linear adsorption
ε	r.m.s. error
ε_b	fluid volume fraction
ε_p	intraparticle void fraction
ξ_o	R_o/R
ρ	density [kg m^{-3}]
ω	angular frequency [rad s^{-1}].

Superscripts

0	zero flow rate
$\hat{}$	amplitude in complex for frequency input
$*$	representative value.

Subscripts

F	fluid
o	outer surface
S	solid
V	volume-mean.

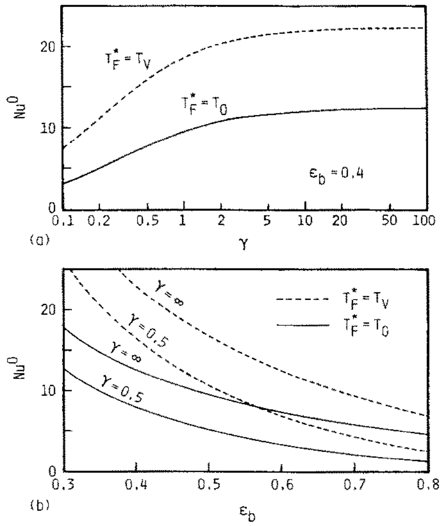

FIG. 1. Limiting Nusselt numbers under the dynamic condition: (a) Nu^0 vs γ at $\varepsilon_b = 0.4$; (b) Nu^0 vs ε_b at $\gamma = 0.5$ and ∞.

In fact, equations (9) and (10) agree with the expressions obtained by Miyauchi [2] for the limiting steady-state Nusselt numbers.

A similar development may be made for mass transfer according to the heat–mass analogy. For example, dynamic limiting Sherwood numbers for a linear adsorption system [7] are obtained from equations (7) and (8) by replacing γ by $\varepsilon_p+\rho_S K_A$. It can be easily shown that, when there is heat generation/sink (or production/disappearance of species by chemical reaction) within the particle, limiting Nusselt (or Sherwood) numbers under the dynamic condition coincide with those under the steady-state condition.

DIFFERENCE BETWEEN STEADY-STATE AND DYNAMIC NUSSELT NUMBERS AT LOW FLOW RATES

In this section, a computer simulation is conducted with a model system to examine any difference between the Nusselt numbers under steady-state and dynamic conditions. The difference is large when γ is small, i.e. solid liquid system.

Suppose that a heat pulse is imposed on water flowing into a packed column of glass beads. The conditions and physical properties used for the calculation are listed in Table 1. When the flow rate is in the laminar flow region, the Nusselt number for this system is expected to agree with the limiting value. The values of $\widehat{Nu}^0$ calculated are shown in Fig. 2 as a function of ω. The response signals can be predicted from the dispersion–concentric model [8]. Angular frequency of harmonics contained in the response signal is less than 0.3 rad s^{-1}. Figure 2 indicates that the limiting Nusselt number for this system may be regarded as constant in this frequency range. Therefore, the limiting Nusselt numbers under the dynamic condition are found to be $Nu^0 = 7$ with $T_F^* = T_o$ and $Nu^0 = 15$ with $T_F^* = T_V$. On the other hand, the limiting Nusselt values under the steady-state condition are 12.5 and 22.5 when $T_F^* = T_o$ and T_V, respectively.

Table 1. Data used for the prediction of temperature

Fluid, water:	density	1000 kg m^{-3}
	specific heat	4180 J kg^{-1} K^{-1}
	thermal conductivity	0.586 W m^{-1} K^{-1}
Particle, glass beads:	radius	0.002 m
	density	2500 kg m^{-3}
	specific heat	670 J kg^{-1} K^{-1}
	thermal conductivity	0.88 W m^{-1} K^{-1}
Column length:		0.01 m
Bed void fraction:		0.4
Interstitial fluid velocity:		0.625×10^{-3} m s^{-1}; $Re = 1$
Axial thermal dispersion coefficient:		0.46×10^{-6} m^2 s^{-1}

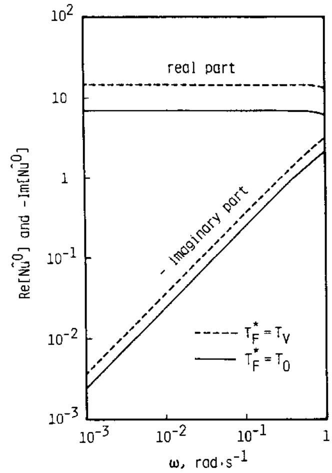

FIG. 2. Frequency dependence of $\widehat{Nu}^0$ for the glass beads–water system. Physical properties are given in Table 1.

The response curves calculated from equilibrium and dynamic Nusselt numbers are illustrated in Figs. 3(a) and (b), respectively, in the cases of $T_F^* = T_o$ and T_V. The difference between the response curves calculated with dynamic and steady-state Nusselt numbers, is evaluated in terms of the r.m.s. error, ε. Since fluid temperature T_F^* is between T_o and T_V, it is expected that careful measurements will enable us to distinguish the steady-state and dynamic Nusselt numbers.

When the flow rate decreases, the response signals calculated with dynamic Nusselt numbers approach those with steady-state values. Therefore, it becomes difficult to detect the difference between dynamic and steady-state Nusselt numbers. For large values of γ for the gas–solid system, dynamic Nusselt numbers approach steady-state values at low flow rates.

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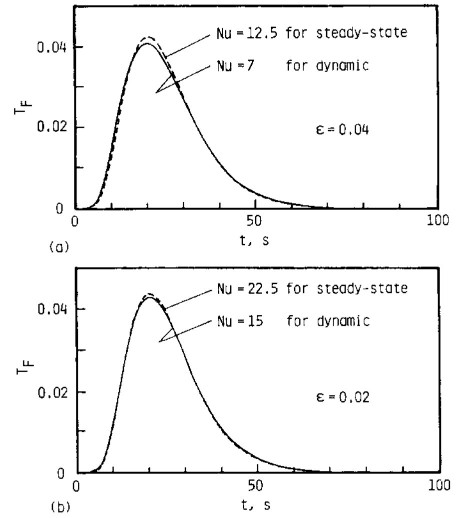

FIG. 3. Comparison of impulse response curves calculated with steady-state and dynamic Nusselt numbers (curves are normalized to have unitary area, $\int_0^\infty T_F\,dt = 1$ s): (a) $T_F^* = T_o$; (b) $T_F^* = T_V$.

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